Model Paper

B.Sc (III-Year) Examination

(New, *w.e.f* 2010-2011)

Mathematics, Paper-IV

(Numerical Analysis)

Time: 3 Hours **Max.Marks:100**

Part-I (Marks: 6x6=36)

Answer any **six** questions

1. Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ using bisection method. Also compute error tolerance percentage at each iteration.

2. Compute a real root of the equation $e^x = x^2$ to an accuracy of 10^{-5} using iterative method.

3. Find the cubic polynomial which takes the following values: $y(0) = 1,\ y(1) = 0,\ y(2) = 1,\ y(3) = 10$ using Newtonôs forward interpolation formula.

4. Find a polynomial $f(x)$ which takes the following values using Newtonôs divided difference formula:

x :	-1	0	3	6	7
$f(x)$:	3	-6	39	822	1611

5. Fit a straight line of the form $y = a_0 + a_1 x$ to the data:

x :	1	2	3	4	6	8
y:	2.4	3.1	3.5	4.2	5.0	6.0

6. From the following table find the area bounded by the curve $f(x)$ and the x-axis from $x = 7.47$ to $x = 7.52$ using Trapezoidal rule:

x :	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$:	1.93	1.95	1.98	2.01	2.03	2.06

7. Solve the following system of equations using Matrix inversion method:

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

8. Given $\frac{dy}{dx} = 1 + xy$ and $y(0) = 1$, obtain Taylor series for $y(x)$ and thereby compute $y(0.1)$ correct to four decimal places.

Part-II (Marks: 4x16=64)

Answer **four** questions, choosing one question from each section

Section – A

9 (a). Establish sufficient condition for a sequence of iterations converges to a root in the case of iteration method.

(b). Solve the equation $x^3 + x^2 + x + 7 = 0$ using False position method.

10 (a). Using Ramanujanôs method, obtain the first eight convergents of the equation $x + x^3 = 1$.

(b). Establish the formula $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$ and hence compute the value of $\sqrt{2}$ to six decimal places.

Section – B

11 . Derive the formula for Gauss forward interpolation formula

12 (a). Given the table of values

x :	50	52	54	56
$f(x)$:	3.684	3.732	3.779	3.825

Find the value of $f(55)$ using Newtonôs backward interpolation formula.

(b). Find the value of $f(304)$ using Lagrangeôs interpolation formula given that

$f(300) = 2.4771,\quad f(304) = 2.4829,\quad f(305) = 2.4843,\quad$ and $f(307) = 2.4871.$

Section – C

13 (a) . Derive the normal equations to fit a curve of the form $y = e^{ax+b}$ by the method of least squares.

(b). Find the values of a, b, c so that $y = a + bx + cx^2$ is the best fit to the data:

x :	0	1	2	3	4
$f(x)$:	1	0	3	10	21

14 (a). Obtain Simpsonôs $\frac{1}{3}$ rule for numerical integration $I = \int_{x_0}^{x_n} y\,dx$

(b). From the following tablular values of x and y obtain $\frac{dy}{dx}$ for $x = 1.2$

x :	1.0	1.2	1.4	1.6	1.8	2	2.0	2.2
y :	2.7183	3.3201	4.0552	4.9532	6.0496	7.3891	9.0250	

Section – D

15. Solve the following system of equations using factorization method:

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8.$$

16. Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ when $x = 0$, find $y(0.2)$, $y(0.4)$, and $y(0.6)$ using Eulerôs method.